Minimization of geometric-beam broadening in a grating-based time-domain delay line for optical coherence tomography application

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This paper discusses a dispersion effect in a grating-based time-domain delay line that is different from the second- or higher-order dispersion in a grating-based Fourier-domain delay line. When the lateral broadening of the beam profile after grating dispersion exceeds the collection aperture of the reference fiber, the peripheral spectrum is decoupled by the fiber. The loss of reference spectral bandwidth by this geometric-beam broadening thus degrades the axial resolution. The polarizing-beam reflector used in the Fourier-domain delay line for suppression of lateral beam walk-off is implemented in this grating-based time-domain delay line to minimize geometric-beam broadening. Theoretical analysis and experiments are given to validate the axial resolution improvement after geometric-beam broadening is minimized. In vitro and in vivo imaging results are presented to demonstrate the improvement. It is also shown that geometric-beam broadening may exist in other optical coherence tomography reference arm configurations. © 2007 Optical Society of America


1. INTRODUCTION
Optical coherence tomography (OCT) [1] emerges as an exciting modality that finds unique application in the diagnosis of disease. The rapid translation of OCT to clinical or preclinical validation is driven by high-speed and high-sensitivity imaging achieved with a variety of system configurations. Time-domain OCT detects the echo interference fringes that are time encoded by either axial-priority reference ranging [2–5] or lateral-priority sample interference fringes that are time encoded by either axial-tem configurations. Time-domain OCT detects the echo high-sensitivity imaging achieved with a variety of sys-tems of how the depth information is obtained, a simple, efficient, and easily aligned optical delay line has a broad appeal not only for the ranging need but also for the depth-tracking application.

Among many types of optical delay lines in OCT, the gratings-based Fourier-domain delay line is perhaps the most accepted one [14–20]. A typical Fourier-domain delay line positions a diffraction grating and a scanning mirror at the conjugate focal planes of an achromatic lens. The tilting of the scanning mirror introduces a linear phase ramp to the dispersed beam, or in other words a wavelength dependent path length $\phi(\omega)$. The first-order derivative of $\phi(\omega)$ is equivalent to a spatial delay in time domain, and the second-order derivative of $\phi(\omega)$ is the group-velocity dispersion. The second- and higher-order dispersions set the limit for optimal axial resolution. The group-velocity dispersion can be canceled by axial offset of the grating [17–19]; however, it is difficult to compensate all higher-order dispersions.

Unlike the Fourier-domain delay line, a time-domain delay line generates wavelength-independent path-length ranging. Therefore an OCT system consisting of a time-domain delay line is free of group-velocity dispersion and higher-order dispersion, even though it may be subject to the dispersion mismatch between reference and sample arms that is common to all OCT setups.

The time-domain delay line is most often constructed by nondispersive optics. Dispersive components such as gratings can also be used to perform time-domain delay [21,22] for OCT applications [23,24]. The grating-based time-domain delay line is shown to have ranging performance (scanning speed, linearity, separate control of phase modulation and optical delay, etc.) comparable to that of the Fourier-domain delay line. Both grating-based time-domain and Fourier-domain delay lines consist of a diffraction grating and a scanning mirror; however the orientation of the grating-mirror pair with respect to the incoming light is flipped in these two setups. The grating-based time-domain delay line generates a direct spatial path-length ranging by scanning the beam laterally upon a Littrow-mounted grating. The grating acts as an “oblique retro reflector” where obliquely incident light is “backreflected.” This configuration produces direct time-domain path-length ranging even though grating is used
as in the Fourier-domain counterpart. The single diffraction loss in grating-based time-domain delay line also makes it more power efficient than the Fourier-domain counterpart, thereby improving signal-to-noise ratio for balanced detection [21].

In this paper we discuss a dispersive effect in grating-based time-domain delay line that is different from group-velocity or higher-order dispersion. This dispersive effect is noted as geometric-beam broadening (GBB), which occurs when the lateral dimension of the beam profile after grating dispersion exceeds the fiber detection aperture. We will then demonstrate that GBB-induced reference spectrum truncation degrades the axial resolution. We will also introduce a polarizing-beam reflector to minimize GBB, and characterize the resulting improvement of axial resolution. Images from in vitro and in vivo biological tissues will be presented to demonstrate the GBB minimization approach. Further discussions on other OCT configurations reveal that GBB is not exclusive to the grating-based time-domain delay line.

2. THEORY

A. Geometric-Beam Broadening in Single-Pass Grating-Based Time-Domain Delay Line

Grating-based time-domain delay line in the original single-pass configuration has been detailed elsewhere [21,22], and it is provided in Fig. 1 for the convenience of addressing the GBB effect.

The solid line in Fig. 1 corresponds to light incident at the axis of scanning mirror, which is also the focal point of the achromatic lens. The generated sector-scanning beam is converted to parallel scanning by the achromatic lens. A Littrow-mounted grating diffracts the beam backward along its incident path. The Littrow mount makes the optical path length change between the mirror pivot and the diffraction point when the mirror scans. This is a depth ranging generated directly in time domain, even though a grating dispersion is used.

Under small angle approximation, the total optical delay length \( l_{\text{scan}} \) is [21]

\[
l_{\text{scan}} = 4f_1\beta_0 \tan \theta_L,
\]

where \( f_1 \) is the achromatic focal length, \( \beta_0 \) is the mirror’s maximum excursion angle, and \( \theta_L \) is the first-order (higher order can be used) Littrow angle of center wavelength \( \lambda_0 \). \( \theta_L \) is determined by

\[
2 \sin \theta_L = \frac{\lambda_0}{p},
\]

where \( p \) is the grating period. The Littrow mount ensures that the \( \lambda_0 \) component will be diffracted exactly along the incoming path; however, since OCT uses a broadband source the spectrum components other than \( \lambda_0 \) are dispersed off the incident direction. The dispersion angle \( \theta_d \) of any spectrum component \( \lambda = \lambda_0 \pm \lambda_d \) with respect to \( \lambda_0 \) is governed by

\[
\sin \theta_L + \sin(\theta_L \pm \theta_d) = \frac{\lambda_0 \pm \lambda_d}{p}.
\]

Small angle approximation of Eq. (3) leads to

\[
\theta_d = \frac{\lambda_d}{p \cos \theta_L}.
\]

The angular dispersion of the beam diffracted from the grating center is collimated by the achromatic lens to an azimuth dimension of

\[
h_d = f_1 \cdot \theta_d = \frac{f_1 \cdot \lambda_d}{p \cos \theta_L}.
\]

This collimated beam is converged to a vertex angle (denoted by \( \theta_{VA} \)) of

\[
\theta_{VA} = \arctan \left( \frac{h_d}{f_2} \right) = \arctan \left( \frac{f_1 \cdot \lambda_d}{f_2 p \cos \theta_L} \right),
\]

at the fiber facet, where \( f_2 \) is the effective focal length of objective lens. Only when this vertex angle is smaller than numerical aperture (NA) (denoted by \( \theta_{NA} \)) of the fiber

\[
\theta_{VA} < \theta_{NA},
\]

can the spectrum component \( \lambda \) be coupled to the fiber. This indicates that decoupling of the peripheral spectrum profile occurs if the beam dimension exceeds the fiber collection aperture. This is noted as GBB. GBB reduces the total spectral width of the reference beam coupled back to the interferometer that in turn degrades the axial resolution of an image.

When the beam is incident at the grating center, the angular dispersion is collimated to a parallel beam. During the galvanometer scanning, however, the off-axis angular dispersion does not collimate to a parallel beam. This may introduce an additional spectrum decoupling if the lateral profile of beam exceeds the dimension of fiber. Figure 2(a) illustrates decoupling of the beam in negative delay with respect to the on-axis beam (the decoupling of the positive delay beam can be analyzed similarly). The beam broadening in Fig. 2(a) for the off-axis beam and at-focus grating is actually equivalent to that in Fig. 2(b) for on-axis beam and off-focus grating. Therefore we will use
the configuration in Fig. 2(b) to investigate GBB because analysis of the on-axis beam is less complicated.

Figure 3 is a modified plot of Fig. 2(b) with the mirror reflection unfolded. The grating offset from the lens focal point is defined as $\Delta f < 0$ for grating moving toward the lens and $\Delta f > 0$ otherwise. It can be derived that (see Appendix A.1)

$$\tan \theta_{VA} = \frac{f_2^2 + (f_2 - l_{og}) \Delta f}{f_2} \tan \theta_d,$$

where $l_{og}$ is the distance between the objective lens and the galvanometer pivot. The inset in Fig. 3 gives the beam geometry at the fiber facet. It is shown that only when $\theta_{VA} < \theta_{NA}$ and the lateral beam profile at the fiber facet is confined within the fiber core can the spectrum component of $\lambda$ be coupled to the fiber. For the case of either $\Delta f < 0$ or $\Delta f > 0$, the maximum wavelength offset $\lambda_d$ for fiber coupling can be calculated by backward ray tracing. The fiber coupling is determined by both $\theta_{NA}$ and $\theta_f$, an angle formed between the fiber core and the virtual converging point of the dispersed beam. Therefore the maximum dispersion angle $\theta'_d$ at the grating for which the spectrum component can be coupled to the fiber is determined by two equations:

If $\theta_{NA} < \theta_f$, the light couples to the fiber when $\theta_{VA} < \theta_{NA}$. This gives

$$\theta'_d = \arctan \left( \frac{f_2 \Delta f}{f_2^2 + (f_2 - l_{og}) \Delta f} \tan \theta_{NA} \right).$$

(9)

If $\theta_{NA} > \theta_f$, the light couples only when $\theta_{VA} < \theta_f$, which gives

$$\theta'_d = \arctan \left( \frac{f_2 \Delta f}{f_2^2 + (f_2 - l_{og}) \Delta f} \tan \theta_f \right),$$

(10)

where

$$\tan \theta_f = \frac{d/2}{f_2} = \frac{f_2^2 + (f_2 - l_{og}) \Delta f}{2f_2^2/\Delta f} d,$$

(11)

for a fiber core diameter $d$.

B. Effect of Geometric-Beam Broadening upon Optical Coherence Tomography Axial Resolution

It is shown that GBB may prevent the peripheral spectrum from coupling back to the OCT system. This decoupling reduces the spectrum bandwidth of the reference light that in turn degrades the axial image resolution.

The normalized spectral power density of a Gaussian source in Fig. 4(a) is

$$S_0(\lambda) = \frac{2 \sqrt{\ln 2/\pi}}{\Delta \lambda_0} \exp \left[ - \frac{4 \ln 2}{\Delta \lambda_0} \left( \frac{\lambda - \lambda_0}{\Delta \lambda_0} \right)^2 \right],$$

(12)

where $\Delta \lambda_0$ is the full width at half-maximum (FWHM) bandwidth measured in wavelength, and the total bandwidth available after spectrum truncation is determined by

$$\Delta \lambda_{cut} = 2 \rho \cos \theta_L \sin \theta'_d.$$

(13)
The truncated Gaussian spectrum may be represented by a nonnormalized “slimmer” Gaussian spectrum that has the same center spectral density and the same overall spectral power as the truncated spectrum does. This slimmer Gaussian spectrum becomes the original source spectrum as the truncation disappears. This equivalent slimmer spectrum may be expressed by

\[
S_{\text{equ}}(\lambda) = \frac{2 \ln 2/\pi}{\Delta \lambda_0} \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_0}{\Delta \lambda_{\text{equ}}} \right)^2 \right],
\]

and the equivalent FWHM bandwidth \( \Delta \lambda_{\text{equ}} \) is determined by (see Appendix A.2 for the derivation):

\[
\Delta \lambda_{\text{equ}} = \Delta \lambda_0 \cdot \text{erf} \left( \sqrt{\ln 2} \cdot \frac{\Delta \lambda_{\text{cut}}}{\Delta \lambda_0} \right),
\]

where \( \text{erf} \) is the error function of

\[
\text{erf}(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^\lambda \exp(-x^2)dx.
\]

The equivalent degraded coherence length of the reference beam is

\[
\rho_{\text{deg}} = \frac{2 \ln 2}{\pi} \cdot \frac{\lambda_0^2}{\Delta \lambda_{\text{equ}}} = \frac{\rho_{\text{opt}}}{\text{erf} \left( \sqrt{\ln 2} \cdot \frac{\Delta \lambda_{\text{cut}}}{\Delta \lambda_0} \right)},
\]

where

\[
\rho_{\text{opt}} = \frac{2 \ln 2}{\pi} \cdot \frac{\lambda_0^2}{\Delta \lambda_0},
\]

is the coherence length of the intact source spectrum or equivalently the optimum axial resolution [25]. Equation (17) is plotted in Fig. 4(b) to quantify resolution degradation \( \rho_{\text{deg}}/\rho_{\text{opt}} \) versus spectrum truncation \( \Delta \lambda_{\text{cut}}/\Delta \lambda_0 \). Significant resolution degradation may be observed for a broadband source if only a small portion of the source spectrum is coupled back to the reference arm.

C. Use of Polarizing-Beam Reflector for Geometric-Beam Broadening Compensation

In a standard Fourier-domain delay line, the dispersed beam recombines to a pencil beam parallel to the incident axis after the second grating diffraction. However, the mirror scanning causes the recombined beam to displace laterally and to walk-off from the fiber coupling aperture. In the grating-based time-domain delay line the diffracted beam is always centered at the incident axis. What GBB introduces is the decoupling of the peripheral beam as the beam lateral dimension varies during the scanning. GBB is thus different from Fourier-domain beam walk-off where the scanning also generates second- and higher-order dispersion.

A polarizing-beam reflector (PBR) consisting of a polarizing beam splitter, a quarter-wave-plate, and a mirror has been used in Fourier-domain delay line to suppress the beam walk-off [20]. This PBR is also useful for GBB compensation. The situation of adding PBR for on-axis and at-focus beam is depicted in Fig. 5, where the entire spectral profile is coupled to the fiber. Thus optimum axial resolution could be achieved for on-axis and at-focus beam if not considering the dispersion mismatch between reference and source arms. Implementing PBR also makes a “double-pass” setup as in Fourier-domain delay line where the mirror excursion is reduced to half for the same amount of depth ranging.

D. Geometric-Beam Broadening in Double-Pass Grating-Based Time-Domain Delay Line

The experimental results in Section 3 show that GBB does occur for on-axis and at-focus beam in single-pass grating-based time-domain delay line. This on-axis and at-focus GBB can be completely suppressed by use of the PBR. Unfortunately PBR does not eliminate the on-axis and off-focus GBB. This is illustrated in Fig. 6(a) for a negative grating offset (similar ray tracing can be ob-
The multielement optics in Fig. 6(a) is unfolded in Figs. 6(b) and 6(c) for explicit ray-tracing analysis: Fig. 6(b) is from the first grating dispersion to PBR mirror reflection, and Fig. 6(c) is from the second grating dispersion to fiber facet coupling. Following similar derivations in Subsection 2.B, the equivalent dispersion angle can be found as (see Appendix A 3 for the details):

\[
\theta'_d \approx \arctan \left\{ \frac{f_1 f_2}{2|\Delta f|\cos \theta_L (l_2 + l_3) [f_1^2 + (l_1 + l_2 - f_2)(f_1 - ku)]} \times \tan \theta_NA \right\},
\]

(19)

where \( k \) is an empirical scaling factor that will be addressed in Subsection 3.C. In Eq. (19) \( u \) is the distance between the achromatic lens and the virtual image after the second grating diffraction [shown in Fig. 6(d)]:

\[
u = f_1 + \Delta f - \frac{2|\Delta f| \cos \beta \sin \theta'}{\sin(\theta_L - \beta) \cos(\theta_L - \text{sgn}(\Delta f) \cdot \theta')} ,
\]

(20)

where

\[
\beta = \theta_L - |\theta' - \theta_d|,
\]

(21)

and

\[
\theta_L \approx \frac{\arctan \left\{ \frac{\tan \theta_NA}{\frac{f_1 f_2}{2|\Delta f|\cos \theta_L (l_2 + l_3) [f_1^2 + (l_1 + l_2 - f_2)(f_1 - ku)]} \times \tan \theta_NA} \right\}}{	heta_NA},
\]

(22)
\[
\theta' = \arctan \left[ \left( 1 - \frac{2l_2 + 2l_3}{f_1^2} \Delta f \right) \tan \theta_0 \right].
\] (22)

Again, \( \Delta f < 0 \) denotes negative offset of grating, and \( \Delta f > 0 \) otherwise. The degradation of axial resolution is then estimated by Eqs. (13)–(17).

3. AXIAL RESOLUTION IMPROVEMENT AFTER GEOMETRIC-BEAM BROADENING MINIMIZATION

A. Axial Image Resolution Measurement

The OCT system was used to perform balanced detection [22–24]. The delay line scanning was synchronized digitally with data acquisition to maintain identical timing for each scan. The sample arm had a mirror mounted on a precision linear stage. The interference fringe was recorded before and after the mirror displacement of 50 \( \mu \)m. Axial resolution was measured by FWHM of interference fringes, as shown in Fig. 7. The phase modulation for fringe measurement was produced by a small offset of the scanning mirror [22]. The interference fringes were first Hilbert transformed then precisely overlapped to extract the number of pixels between two envelope peaks. This number corresponds to 50 \( \mu \)m, and it was then used to calculate FWHM of the fringe envelopes at two mirror positions. The mean value of two FWHMs was taken as the axial image resolution.

B. Image Resolution of the Single-Pass Setup

The source used in the OCT system was a superluminescent diode of 1300 nm with 40 nm bandwidth giving 18.6 \( \mu \)m coherence length in the air. The achromatic lens had a focal length of 50.8 mm, and the objective lens had an effective focal length of 3.4 mm. The single-mode fiber had a core diameter of 9 \( \mu \)m and NA of 0.13. The resolution measurements and simulation results for single-pass setup are shown in Fig. 8, where the best resolution of 28 \( \mu \)m appears to be present at the off-focus grating position. There were relatively sparse experimental data points; however, simulation results agree with the measurement pattern.

C. Image Resolution of the Double-Pass Setup

The theoretical axial resolution and corresponding measurements of double-pass setup are shown in Fig. 9(a). The calculation is based on Eq. (19) by setting \( k = -1.3 \). Ideally \( k = 1 \) for the incidence of a perfect pencil beam. However, the Gaussian profile of the beam will likely introduce error into ray tracing in the double-pass configuration, as it involves many reflections and diffractions. This error may be corrected by an empirical scaling parameter \( k \) in Eq. (19) as evidenced by the good agreement between simulation and measurements at \( k = -1.3 \). The \( k \) is an empirical term; therefore its value is dependent on the delay line parameters, such as focal length, grating period, and center wavelength. This scaling factor is perhaps unnecessary in single-pass calculation, where only one reflection exists. The inclusion of \( k \) is further supported by measurements of the signal intensity. The overall spectral power coupled back to the fiber decreases when GBB exists, and the spectral power linearly determines the signal intensity in balanced detection. Figure 9(b) plots the measured signal intensity and the calculated ratio of coupled spectral power to the entire source power. The ratio calculation is based on:

\[
P_r = \frac{\int_{\lambda_0 - (\Delta \lambda_{cut}/2)}^{\lambda_0 + (\Delta \lambda_{cut}/2)} 2\sqrt{\ln 2/\pi} \frac{\Delta \lambda_0}{\lambda_0} \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_0}{\Delta \lambda_0} \right)^2 \right] d\lambda}{\frac{\sqrt{\ln 2 \cdot \Delta \lambda_{cut}}}{\Delta \lambda_0}}.
\] (23)

The good agreement between theory and measurement in both Figs. 9(a) and 9(b) where the same value of \( k \) is used validates inclusion of an empirical scaling factor in the calculation.

Unlike in single pass, the double-pass setup optimizes the axial resolution for an at-focus grating. The optimized resolution of 19 \( \mu \)m is also very close to the source coherence length. 

![Fig. 7. (Color online) Axial image resolution measurement. Interference signals at target separation of 50 \( \mu \)m are overlapped to calibrate the pixel size, which in turn is used to calculate FWHM of the intensity envelope profile.](image-url)
ence length. It is observed in Fig. 9(a) that the change of resolution as a function of grating offset in double pass seems to be much steeper than that of single pass. It is important to point out that resolution change in double pass should be interpreted by doubling the scan range for the same amount of grating offset. The grating offset of ±0.625 mm corresponds to 1.25 mm depth ranging in single pass and 2.5 mm depth ranging in double pass. Within this 2.5 mm ranging depth, which is also typical for OCT imaging of highly scattering biological tissue, it is clear that the resolution in double pass is much better than in single pass.

Figure 9(a) represents on-axis scanning of an off-focus grating. In the actual OCT imaging the scanning is performed by off-axis scanning of an at-focus grating. Figure 9(c) represents the resolution measured for off-axis and at-focus scanning. The 2.5 mm range in Fig. 9(c) also corresponds to the grating offset of ±0.625 mm in Fig. 9(a). It has been illustrated previously that off-axis and at-focus scanning is equivalent to on-axis and off-focus scanning. The theoretical calculation in Fig. 9(c), which is based on on-axis and off-focus beams, agrees with the off-axis and at-focus measurement except for slight offset at the edges. Figure 9(c) demonstrates that double pass renders axial image resolution of 19–25 μm throughout a 2.5 mm depth ranging for a source coherence length of 18.6 μm.

4. IMAGING RESULTS

A. In Vivo Imaging of Human Nail Structure before and after Geometric-Beam Broadening Minimization

A finger nail from a human volunteer was imaged before and after GBB minimization. The sample arm has been altered to use a gradient-index lens probe. Shown in the top row of Fig. 10 is the anatomy. OCT images were taken at the three marked regions: one through the proximal nail fold [12], one through the middle part of the nail plate, and one through the distal groove. The images in the center row were taken without GBB minimization, and the images in the bottom row were taken with GBB minimization. Both were from the same subject, however, at different nail lengths. Therefore some mismatch of the distal nail plate can be observed between two sets of images. Both sets of images give good anatomic information, whereas the double-pass one has a much finer appearance and thinner yet clearer tissue interfaces. The finer appearance may be the result of a finer lateral resolution if there was any; however, it is believed that the improvement of axial resolution contributes to the improved layer delineation.

B. In Vitro Imaging of Porcine Coronary Artery

A freshly excised porcine coronary artery was imaged by double-pass configuration at different resolution settings. The resolution was administered by axial offset of grating, and the resolution setting for each grating offset is deduced from Fig. 9(a). This number corresponds to the best possible resolution for a specific grating offset. The images in Fig. 11 were taken at three resolution settings: 65, 30, and 19 μm. The artery was dissected to open up the inner surface of the lumen, and the saline solution used for dehydration can be seen in the image. A histological slice of the coronary artery is displayed to correlate the three layers of intima, media, and adventitia. The three layers are identified in all these images, whereas the layer transition is less blurred in the 19 μm resolution. This can only be explained with axial resolution improvement.

5. DISCUSSION

GBB in grating-based time-domain delay line is caused by decollimation of dispersed beam. Decoupling of the pe-
Fig. 10. (Color online) *In vivo* imaging of a human nail. The upper row was taken at single pass and the bottom row was taken at double pass. The nail anatomy is adapted from www.footdoc.ca.

Fig. 11. *In vitro* imaging of a porcine coronary artery at different axial resolution settings. Histology reference is given at lower right.
Peripheral spectral profile occurs when beam broadening exceeds the detection aperture. This is different from the beam walk-off in the Fourier-domain delay line where entire beam may be decoupled. GBB is not seen in most Fourier-domain delay lines; however, it does occur in a transmissive Fourier-domain delay line and a nonscanning full-field OCT setup based on Littrow-mounted grating.

The recently reported transmissive Fourier-domain delay line [26,27] is drawn in Fig. 12(a) by adding the ray tracing for the dispersed beam. It is necessary in this configuration to offset the grating from the lens focal point to suppress the group-velocity dispersion [26]. However GBB occurs whenever there is grating offset in this delay line. GBB in this delay line is visualized in Fig. 12(a) by the dotted and dashed lines. Assuming the beam after M2 reflection and the outgoing beam are parallel to the optical axis to simplify the analysis, the angle formed by the dispersed beam before the second grating diffraction is found to be

\[
\theta_2 = \arctan \left( \frac{2 \cos 2\beta \cdot \tan \theta_d}{1 + \cos 4\beta - (1 - \cos 4\beta) \frac{\Delta f^2}{f^2} \tan^2 \theta_d} \right).
\]

where \(\beta\) is the mirror tilting angle. GBB at the outlet lens becomes

\[
h_\delta = \left( \frac{1}{\cos 2\beta} \right) \Delta f \cdot \cos \theta_g \cdot \tan \theta_d = \left( \frac{1}{\cos 2\beta} \right) \Delta f \cdot \cos \theta_g \cdot 2 \cos 2\beta \cdot \tan \theta_d = \frac{\Delta f^2}{f^2} \tan^2 \theta_d = 2\Delta f \theta_d \cos \theta_g.
\]

Using PBR to minimize GBB requires that there is a common but opposite path for the incoming and returning beam. In this transmissive Fourier-domain delay line the incoming and outgoing beam paths are separated; therefore, a PBR will not be useful for GBB minimization. It is likely that GBB cannot be compensated if the transmission configuration is maintained.

The Littrow-mounted grating for nonscanning full-field OCT [12,28,29] is also shown in Fig. 12(b) by adding the ray tracing for the dispersed beam. The grating center is focused at the detector plane; however, the off-center diffracting point is defocused at the detection plane. This generates a GBB of

\[
h_\delta = \frac{2f - \Delta f}{2(f - \Delta f)} \Delta f \cdot \tan \theta_d = \Delta f \cdot \tan \theta_d,
\]

at the detection plane. This GBB will affect the uniformity of both the axial resolution and the sensitivity in lateral dimension. Such an effect could be observed in Fig. 5 in [12].

Using PBR to minimize GBB also requires that the dispersed beam become collimated or nearly collimated after a lens. The configuration in Fig. 12(b) does not have any region where this condition could be satisfied; therefore GBB is likely to remain in this configuration.

6. CONCLUSIONS

In this paper, a geometric dispersion effect in a grating-based time-domain delay line is discussed. This effect refers to the condition of GBB when the beam lateral dimension exceeds collection aperture of the reference fiber. This is different from the group-velocity dispersion in grating-based Fourier-domain delay line. GBB gives rise to decoupling of the peripheral spectral profile. Axial resolution is thus expected to degrade due to the equivalent narrowing of the reference spectral bandwidth. The PBR used for walk-off suppression in Fourier-domain delay line is implemented in this grating-based time-domain delay line to minimize GBB. It is demonstrated that the optimum axial resolution for on-axis and at-focus beams can be recovered, and the axial resolution for off-axis and at-focus beams within a typical OCT ranging depth can be improved significantly. It is also shown that a similar GBB effect can be found in some other OCT reference arm configurations where the compensation method remains an open problem.
APPENDIX A

1. Single-Pass Grating-Based Time-Domain Delay Line
This section derives the relationship between dispersion angle $\theta_d$ at the grating and beam convergence angle $\theta_{VA}$ at the fiber facet for on-axis and off-focus conditions in Fig. 3.

For $\Delta f < 0$,
\[ h_d = \left( \frac{f_1^2}{\Delta f} + l_{og} \right) \cdot \tan \theta_1 = \left( \frac{f_1^2}{\Delta f} + l_{og} \right) \cdot \frac{\Delta f}{f_1} \tan \theta_d, \quad (A1) \]

where $h_d$ is half of the beam width at the objective lens and $\theta_1$ is the angle between the optical axis of the achromatic lens and the dispersed beam after lens collimation. Equation (A1) leads to
\[ \tan \theta_{VA} = \frac{h_d}{\Delta x + f_2} = \frac{f_1^2 + (f_2 - l_{og})\Delta f}{f_2} \tan \theta_d, \quad (A2) \]

where $\Delta x$ is the offset of the virtual imaging point from the fiber facet.

Similar derivations for $\Delta f > 0$ give
\[ h_d = \left( \frac{f_1^2}{\Delta f} - l_{og} \right) \cdot \frac{\Delta f}{f_1} \tan \theta_d, \quad (A3) \]
\[ \tan \theta_{VA} = \frac{h_d}{f_2 - \Delta x} = \frac{f_1^2 + (f_2 - l_{og})\Delta f}{f_2} \tan \theta_d. \quad (A4) \]

2. Spectrum Truncation and Axial Resolution Degradation
The total bandwidth $\Delta \lambda_{cut}$ of truncated spectrum is determined by equivalent beam dispersion angle $\theta_d'$:
\[ \Delta \lambda_{cut} = 2p \cos \theta_d' \sin \theta_d'. \quad (A5) \]

The spectral power of the truncated spectrum is found by
\[ SP_{cut} = \int_{\lambda_0-(\Delta \lambda_{cut}/2)}^{\lambda_0+(\Delta \lambda_{cut}/2)} S_{\lambda}(\lambda) \cdot d\lambda = 2 \sqrt{\frac{2\ln 2}{\pi}} \frac{\Delta \lambda_{cut}}{\Delta \lambda_0} \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_0}{\Delta \lambda_0} \right)^2 \right] \cdot d\lambda. \quad (A6) \]

Assuming $\lambda' = 2\sqrt{\ln 2}(\lambda - \lambda_0)/\Delta \lambda_0$, then
\[ SP_{cut} = \frac{2}{\sqrt{\pi}} \int_0^{\ln \frac{\Delta \lambda_{cut}}{\Delta \lambda_0}} \exp(-\lambda'^2) d\lambda' = \text{erf} \left( \frac{\ln 2 \Delta \lambda_{cut}}{\Delta \lambda_0} \right). \quad (A7) \]

The equivalent slimmer Gaussian spectrum is defined as to have the same spectral power at the center wavelength and the total spectral power as the truncated spectrum does. This gives
\[ S_{eq}(\lambda) = \frac{\Delta \lambda_{eq}}{\Delta \lambda_0} \cdot \frac{2 \sqrt{2 \ln 2 \pi}}{\Delta \lambda_{eq}} \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_0}{\Delta \lambda_{eq}} \right)^2 \right], \quad (A8) \]

\[ \int_0^\infty S_{eq}(\lambda) d\lambda = \frac{\Delta \lambda_{eq}}{\Delta \lambda_0} = SP_{cut}(\lambda) = \text{erf} \left( \frac{\sqrt{2} \Delta \lambda_{cut}}{\Delta \lambda_0} \right). \quad (A9) \]
\[ \Delta \lambda_{eq} = \Delta \lambda_0 \cdot \text{erf} \left( \frac{\sqrt{2} \Delta \lambda_{cut}}{\Delta \lambda_0} \right). \quad (A10) \]

3. Double-Pass Grating-Based Time-Domain Delay Line
For the double-pass configuration using PBR, the ray tracing can be unfolded to two segments. Figure 6(b) is from the first grating dispersion to reflection at the scanning mirror, and Fig. 6(c) is from the second grating dispersion to fiber coupling. $l_1$, $l_2$, and $l_3$ are distances from the PBR to the fiber, mirror, and galvanometer. $\alpha$ and $\beta$ are incident and diffracting angles of the beam for the second grating diffraction. The equivalent angular dispersion of fiber coupling is determined by
\[ \tan \theta' = \left( 1 - \frac{2l_2 + 2l_3}{f_1^2 \Delta f} \right) \tan \theta_d, \quad (A11) \]
\[ \alpha = \theta_L + \theta' = \theta_L + \arctan \left[ 1 - \frac{2l_2 + 2l_3}{f_1^2 \Delta f} \right] \tan \theta_L - \theta'. \quad (A12) \]

The grating equation of
\[ p(\sin \alpha + \sin \beta) = \lambda_0 + \frac{\Delta \lambda_0}{2} = p[\sin \theta_L + \sin(\theta_L + \theta_d)], \quad (A13) \]
gives
\[ \beta = \arcsin[\sin \theta_L + \sin(\theta_L + \theta_d) - \sin(\theta_L + \theta' - \theta)]. \quad (A14) \]

When $\Delta f < 0$ and if $\theta' > \theta_d$, one has $\beta < \theta_d$, therefore
\[ \tan \theta_{VA} = \frac{f_1^2 + (l_1 + l_2 - f_2)(f_1 - u)}{f_2} \tan(\theta_L - \beta), \quad (A15) \]

where
\[ u = f_1 + \Delta f - \frac{2 \cos \beta \sin \theta'}{\sin(\theta_L - \beta) \cos(\theta_L + \theta') \Delta f}. \quad (A16) \]

Averaging the spectral components symmetric to the center wavelength gives
\[ \tan(\theta_L - \beta) \approx \frac{\sin(\theta_L - \theta_d) - \sin(\theta_L - \theta') - \sin(\theta_L + \theta_d) + \sin(\theta_L + \theta')}{2} = \cos \theta_L (\tan \theta' - \tan \theta_d). \quad (A17) \]

Equations (A11), (A15), and (A17) lead to...
\tan \theta_2 = \frac{f_2^3}{2\Delta f \cdot \cos(\theta_2) \cdot (l_2 + l_3)[f_1^2 + (l_1 + l_2 - f_2)(f_1 - ku)]} \times \tan \theta_{\text{LA}}, \quad \text{(A18)}

where \( k \) is the empirical scaling factor discussed in Subsection 3.C.

**REFERENCES**


